*International Journal of Theoretical Physics, Vol. 30, No. 7, 1991*

# Higgs-Field Gravity within the Standard Model

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*Received July 18, 1990*

Within the framework of the Glashow–Salam–Weinberg model it is shown that the Higgs field mediates an attractive scalar gravitational interaction of Yukawa type between the elementary particles which become massive by the ground state of the Higgs field after symmetry breaking.

## 1. INTRODUCTION

Until now the origin of the mass of the elementary particles has been unclear. Usually mass is introduced by the interaction with the Higgs field; however, in this way the mass is not explained, but only reduced to the parameters of the Higgs potential, whereby the physical meaning of the Higgs field and its potential remains nonunderstood.

On the other hand, there exists an old idea of Einstein, the so-called "principle of relativity of inertia," according to which mass should be produced by the interaction with the gravitational field (Einstein, 1917). Einstein argued that the inertial mass is only a measure for the resistance of a particle against the relative acceleration with respect to other particles; therefore, within a consequent theory of relativity, the mass of a particle should be originated by interaction with all other particles of the universe, whereby this interaction should be the gravitational one which couples to all particles, i.e., to their masses or energies. He even postulated that the value of the mass of a particle should go to zero if one puts the particle at an infinite distance from all others.

This fascinating idea was not very successful within Einstein's theory of gravity, i.e., general relativity, although it caused Einstein to introduce the cosmological constant in order to construct a cosmological model with finite space, and led Brans and Dicke (1961) to develop their scalar-tensor theory. But an explanation of the mass has not followed from it until now.

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0020-7748/91/0700-0985$06.50/0 

In this paper we will show that a successful Higgs-field mechanism lies in the direction of Einstein's idea of producing mass by gravitational interaction; we find that the Higgs field as source of the inertial mass is related to gravity (Dehnen *et al.*, 1990), i.e., it mediates a scalar gravitational interaction between the massive particles, however, of Yukawa type. This results from the fact that the Higgs field itself becomes massive after symmetry breaking. On the other hand, an estimation of the coupling constants shows that it may be improbable that this Higgs-field gravity can be identified with any experimental evidence. Perhaps its applicability lies beyond the scope of present experimental capabilities.

## 2. GRAVITATIONAL ACTION OF THE HIGGS FIELD

In a previous publication (Dehnen *et al.*, 1990) we have shown approximatively the gravitational interaction of the Higgs field between massive fermions. In the present paper we extend our investigation in an exact manner to fermions and bosons. Due to this reason we perform our calculations within the well-established Glashow–Salam–Weinberg model of electroweak interaction based on the localized group $SU(2)\times U(1)$, taking into account all families of elementary particles. For this we start with the following definitions (spinor and isospin indices are suppressed):

$$\psi^i = \psi^{m_i} = \begin{pmatrix} \psi^{l_i} \\ \psi^{q_i} \end{pmatrix}, \qquad m = l, q \tag{2.1}$$

represents the spinorial wave functions of the $i$th family ($i = 1, \ldots, N_f$), wherein

$$\psi^{l_i} = \psi^{l_i}_L + \psi^{l_i}_R \tag{2.2}$$

is the leptonic part with

$$\psi^{l_i}_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix}, \qquad \psi^{l_i}_R = e^i_R \tag{2.2a}$$

and

$$\psi^{q_i} = \psi^{q_i}_L + \psi^{q_i}_R \tag{2.3}$$

means the part of the quarks with

$$\psi^{q_i}_L = \begin{pmatrix} u^i_L \\ d'^i_L \end{pmatrix}, \qquad \psi^{q_i}_R = \begin{pmatrix} u^i_R \\ d'^i_R \end{pmatrix} \tag{2.3a}$$

and

$$d'^i = U^i_{(c)j} d^j \tag{2.3b}$$

as the Cabibbo transformed quark wave functions. Here the left-handed fermions $\psi_L^{l_i}$ and $\psi_L^{q_i}$ are doublets with respect to the localized group $SU(2)$, whereas the right-handed ones $\psi_R^{l_i}$ and $\psi_R^{q_i}$ are singlets. Correspondingly, the covariant derivatives take the form

$$\begin{aligned}
D_\lambda\psi_L^{l_i} &= (\partial_\lambda + ig_2 W_\lambda^a\tau_a - \tfrac{1}{2}ig_1 B_\lambda)\psi_L^{l_i} \\
D_\lambda\psi_L^{q_i} &= (\partial_\lambda + ig_2 W_\lambda^a\tau_a + \tfrac{1}{6}ig_1 B_\lambda)\psi_L^{q_i} \\
D_\lambda\psi_R^{l_i} &= (\partial_\lambda - ig_1 B_\lambda)\psi_R^{l_i} \\
D_\lambda\psi_R^{q_i} &= D_\lambda\begin{pmatrix} u_R^i \\ d_R'^i \end{pmatrix} = \begin{cases}(\partial_\lambda + \tfrac{2}{3}ig_1 B_\lambda)u_R^i \\ (\partial_\lambda - \tfrac{1}{3}ig_1 B_\lambda)d_R'^i\end{cases}
\end{aligned} \tag{2.4}$$

Herein $\tau^a$ are the generators of the group $SU(2)$, $W_\lambda^a$ represent the corresponding gauge potentials, and $B_\lambda$ is the $U(1)$ gauge potential with $g_1$ and $g_2$ as gauge-coupling constants. The covariant gauge-field strengths are given by the commutators

$$\begin{aligned}
\mathcal{F}_{(2)\mu\nu} &= F^a_{(2)\mu\nu}\tau_a = \frac{1}{ig_2}[D_\mu^{(2)}, D_\nu^{(2)}] \\
\mathcal{F}_{(1)\mu\nu} &= F_{(1)\mu\nu}Y = \frac{1}{ig_1}[D_\mu^{(1)}, D_\nu^{(1)}]
\end{aligned} \tag{2.5}$$

[(1) and (2) refer to the groups $U(1)$ and $SU(2)$, respectively]. Here $Y$ is the $U(1)$ generator of the weak hypercharge in the different representations according to (2.4), where we follow the notation of Saller (1985) and not of Becher *et al.* (1981). Finally, we introduce a scalar Higgs field $\phi$ belonging to the fundamental representation of $SU(2)$; its covariant derivative reads

$$D_\lambda\phi = (\partial_\lambda + ig_2 W_\lambda^a\tau_a + \tfrac{1}{2}ig_1 B_\lambda)\phi \tag{2.6}$$

Herewith we construct the gauge invariant minimally coupled Lagrange density:

$$L = L(\psi) + L(F) + L(\phi) \tag{2.7}$$

where

$$L(\psi) = i\frac{\hbar}{2}[\bar\psi_{Lm_i}\gamma^\lambda D_\lambda\psi_L^{m_i} + \bar\psi_{Rm_i}\gamma^\lambda D_\lambda\psi_R^{m_i}] + \text{h.c.} \tag{2.7a}$$

$$L(F) = -\frac{\hbar}{16\pi}(F^a_{(2)\lambda\mu}F^{\lambda\mu}_{(2)a} + F_{(1)\lambda\mu}F^{\lambda\mu}_{(1)}) \tag{2.7b}$$

and

$$L(\phi) = \frac{1}{2}(D_\lambda\phi)^\dagger D^\lambda\phi - \frac{\mu^2}{2}\phi^\dagger\phi - \frac{\lambda}{4!}(\phi^\dagger\phi)^2 - k\bar\psi_{Rm_i}\phi^\dagger\hat{x}^{m_i}_{n_j}\psi_L^{n_j} + \text{h.c.} \tag{2.7c}$$

($\mu^2$, $\lambda$, $k$ are real parameters of the Higgs potential and $\hat{x}^{m_i}_{n_j}$ is the Yukawa coupling matrix, see appendix). The field equations following from Hamilton's action principle result in the wave equations for the left- and right-handed fermions,

$$i\gamma^\mu D_\mu \psi^{m_i}_L - \frac{k}{\hbar}\hat{x}^{\dagger m_i}_{n_j}\phi\psi^{n_j}_R = 0 \tag{2.8a}$$

$$i\gamma^\mu D_\mu \psi^{m_i}_R - \frac{k}{\hbar}\phi^\dagger \hat{x}^{m_i}_{n_j}\psi^{n_j}_L = 0 \tag{2.8b}$$

in the Yang–Mills equations

$$D_\mu F^{\mu\lambda}_{(2)a} \equiv \partial_\mu F^{\mu\lambda}_{(2)a} - g_2\varepsilon_{abc}W^b_\mu F^{c\mu\lambda}_{(2)} = 4\pi j^\lambda_{(2)a} \tag{2.9a}$$

$$\partial_\mu F^{\mu\lambda}_{(1)} = 4\pi j^\lambda_{(1)} \tag{2.9b}$$

($\varepsilon_{abc}$ is the Levi-Civita symbol) with the current densities

$$j^\lambda_{(2)a} = g_2\bar{\psi}_{Lm_i}\gamma^\lambda\tau_a\psi^{m_i}_L + i\frac{g_2}{2\hbar}[\phi^\dagger\tau_a D^\lambda\phi - (D^\lambda\phi)^\dagger\tau_a\phi] \tag{2.10a}$$

$$j^\lambda_{(1)} = g_1[Y\bar{\psi}_{Lm_i}\gamma^\lambda\psi^{m_i}_L + Y\bar{\psi}_{Rm_i}\gamma^\lambda\psi^{m_i}_R] + i\frac{g_1}{4\hbar}[\phi^\dagger D^\lambda\phi - (D^\lambda\phi)^\dagger\phi] \tag{2.10b}$$

and in the Higgs-field equation

$$D_\mu D^\mu\phi + \mu^2\phi + \frac{\lambda}{6}(\phi^\dagger\phi)\phi = -2k\bar{\psi}_{Rm_i}\hat{x}^{m_i}_{n_j}\psi^{n_j}_L \tag{2.11}$$

Obviously the current densities separate into two gauge-covariant parts $j^\lambda_{(2)a}(\psi)$ and $j^\lambda_{(2)a}(\phi)$ as well as $j^\lambda_{(1)}(\psi)$ and $j^\lambda_{(1)}(\phi)$. In a similar way the gauge-invariant canonical energy-momentum tensor consists of three gauge-invariant parts:

$$T^\mu_\lambda = T^\mu_\lambda(\psi) + T^\mu_\lambda(F) + T^\mu_\lambda(\phi) \tag{2.12}$$

with

$$T^\mu_\lambda(\psi) = i\frac{\hbar}{2}[\bar{\psi}_{Lm_i}\gamma^\mu D_\lambda\psi^{m_i}_L + \bar{\psi}_{Rm_i}\gamma^\mu D_\lambda\psi^{m_i}_R] + \text{h.c.}$$

$$T^\mu_\lambda(F) = -\frac{\hbar}{4\pi}\left[\left(F^{(a)}_{(2)\lambda\nu}F^{\mu\nu}_{(2)a} - \frac{1}{4}\delta^\mu_\lambda F^a_{(2)\alpha\beta}F^{\alpha\beta}_{(2)a}\right)\right. \tag{2.12a}$$

$$\left. + \left(F_{(1)\lambda\nu}F^{\mu\nu}_{(1)} - \frac{1}{4}\delta^\mu_\lambda F_{(1)\alpha\beta}F^{\alpha\beta}_{(1)}\right)\right] \tag{2.12b}$$

and

$$T^{\mu}_{\lambda}(\phi)=\frac{1}{2}\left[(D_{\lambda}\phi)^{\dagger}D^{\mu}\phi+(D^{\mu}\phi)^{\dagger}D_{\lambda}\phi\right.$$
$$\left.-\delta^{\mu}_{\lambda}\left\{(D_{\alpha}\phi)^{\dagger}D^{\alpha}\phi-\mu^{2}\phi^{\dagger}\phi-\frac{\lambda}{12}(\phi^{\dagger}\phi)^{2}\right\}\right] \tag{2.12c}$$

With respect to the field equations, the conservation laws for energy and momentum of the whole system of fields are valid,

$$\partial_{\mu}T^{\mu}_{\lambda}=0 \tag{2.13}$$

In view of analyzing the interaction caused by the Higgs field, we investigate first the equation of motion for the expectation value of the 4-momentum of the fermionic matter fields ($\psi$ fields) and the gauge fields ($F$ fields). From (2.12) and (2.13) one finds immediately, neglecting surface integrals at spacelike infinity,

$$\partial_{0}\int[T^{0}_{\lambda}(\psi)+T^{0}_{\lambda}(F)]\,d^{3}x=-\int\partial_{\mu}T^{\mu}_{\lambda}(\phi)\,d^{3}x \tag{2.14}$$

Insertion of $T^{\mu}_{\lambda}(\phi)$ according to (2.12c) and elimination of the second derivatives of the Higgs field by the field equation (2.11) results, with the use of the definitions of the field strengths $F_{(1)\mu\nu}$ and $F^{a}_{(2)\mu\nu}$ in

$$\frac{\partial}{\partial t}\int[T^{0}_{\lambda}(\psi)+T^{0}_{\lambda}(F)]\,d^{3}x$$
$$=k\int[(D_{\lambda}\phi)^{\dagger}\bar{\psi}_{Rm_i}\hat{x}^{m_i}_{n_j}\psi^{n_j}_{L}+\bar{\psi}_{Lm_i}\hat{x}^{\dagger m_i}_{n_j}\psi^{n_j}_{R}D_{\lambda}\phi]\,d^{3}x$$
$$+\frac{i}{2}\int\left[g_{2}F^{a}_{(2)\mu\lambda}\{\phi^{\dagger}\tau_{a}D^{\mu}\phi-(D^{\mu}\phi)^{\dagger}\tau_{a}\phi\}\right.$$
$$\left.+\frac{1}{2}g_{1}F_{(1)\mu\lambda}\{\phi^{\dagger}D^{\mu}\phi-(D^{\mu}\phi)^{\dagger}\phi\}\right]d^{3}x \tag{2.15}$$

The right-hand side represents the expectation value of the 4-force, which changes the 4-momentum of the $\psi$ fields and of the $F$ fields with time. However, the latter expression can be rewritten with the use of the field equations (2.9a) and (2.9b) as follows:

$$\partial_{\mu}T^{\mu}_{\lambda}(F)=\hbar[F^{a}_{(2)\mu\lambda}\{j^{\mu}_{(2)a}(\psi)+j^{\mu}_{(2)a}(\phi)\}+F_{(1)\mu\lambda}\{j^{\mu}_{(1)}(\psi)+j^{\mu}_{(1)}(\phi)\}] \tag{2.16}$$

Herewith one obtains instead of (2.15)

$$\frac{\partial}{\partial t}\int T^{0}_{\lambda}(\psi)\,d^{3}x=\int\hbar[F^{a}_{(2)\mu\lambda}j^{\mu}_{(2)a}(\psi)+F_{(1)\mu\lambda}j^{\mu}_{(1)}(\psi)]\,d^{3}x$$
$$+k\int[(D_{\lambda}\phi)^{\dagger}\bar{\psi}_{Rm_i}\hat{x}^{m_i}_{n_j}\psi^{n_j}_{L}+\bar{\psi}_{Lm_i}\hat{x}^{\dagger m_i}_{n_j}\psi^{n_j}_{R}D_{\lambda}\phi]\,d^{3}x \tag{2.17}$$

where on the right-hand side we have the 4-force of the gauge fields and the Higgs field, both acting on the matter field and changing its 4-momentum. Evidently, the gauge field strengths couple to the gauge currents $j^{\mu}_{(2)a}(\psi)$ and $j^{\mu}_{(1)}(\psi)$, i.e., to the gauge-coupling constants $g_1$ and $g_2$ according to (2.10a) and (2.10b), whereas the Higgs field strength (gradient of the Higgs field) couples to the fermionic mass parameter $k$ only (Becher *et al.*, 1981). This fact points to a *gravitational* action of the scalar Higgs field.

## 3. FIELD EQUATIONS OF HIGGS GRAVITY

For demonstrating the gravitational interaction explicitly, we first perform the spontaneous symmetry breaking, because in the case of a scalar gravity only massive particles should interact.[2] For this $\mu^2 < 0$ must be valid, and according to (2.11) and (2.12c), the ground state $\phi_0$ of the Higgs field is defined by

$$\phi_0^{\dagger}\phi_0 = v^2 = \frac{-6\mu^2}{\lambda} \tag{3.1}$$

which we resolve as

$$\phi_0 = vN \tag{3.2}$$

with

$$N^{\dagger}N = 1, \qquad \partial_{\lambda}N = 0 \tag{3.2a}$$

The general Higgs field $\phi$ is different from (3.2) by a local unitary transformation:

$$\phi = \rho UN, \qquad U^{\dagger}U = 1 \tag{3.3}$$

with

$$\phi^{\dagger}\phi = \rho^2, \qquad \rho = v(1+\varphi) \tag{3.3a}$$

where $\varphi$ represents the real-valued excited Higgs field. Now we use the possibility of a unitary gauge transformation which is inverse to (3.3):

$$\phi' = U^{-1}\phi, \qquad \psi' = U^{-1}\psi, \qquad \mathscr{F}'_{\mu\nu} = U^{-1}\mathscr{F}_{\mu\nu}U \tag{3.4}$$

so that

$$\phi' = \rho N \tag{3.4a}$$

[2]The only possible source of a classical scalar gravity is the trace of the energy-momentum tensor.

is valid, and perform in the following all calculations in the gauge (3.4) (unitary gauge).

Using (3.2)-(3.4a), the field equations (2.8a)-(2.11) take the form, avoiding the strokes introduced in (3.4),

$$i\gamma^{\mu}D_{\mu}\psi_{L}^{m_i}-\frac{1}{\hbar}(1+\varphi)\hat{m}_{n_j}^{m_i}\psi_{R}^{n_j}=0 \tag{3.5a}$$

$$i\gamma^{\mu}D_{\mu}\psi_{R}^{m_i}-\frac{1}{\hbar}(1+\varphi)\hat{m}_{n_j}^{m_i}\psi_{L}^{n_j}=0 \tag{3.5b}$$

$$D_{\mu}F_{(2)a}^{\mu\lambda}+\frac{1}{\hbar^2}(1+\varphi)^2[M_{(2)ab}^2W^{b\lambda}+M_{(1,2)a}^2B^{\lambda}]=4\pi j_{(2)a}^{\lambda}(\psi) \tag{3.6a}$$

$$\partial_{\mu}F_{(1)}^{\mu\lambda}+\frac{1}{\hbar^2}(1+\varphi)^2[M_{(1,2)a}^2W^{a\lambda}+M_{(1)}^2B^{\lambda}]=4\pi j_{(1)}^{\lambda}(\psi) \tag{3.6b}$$

$$\begin{aligned}\partial_{\mu}\partial^{\mu}\varphi&+\frac{M^2}{\hbar^2}\varphi+\frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2+\varphi^3)\\ &=-\frac{1}{v^2}\Bigg[\bar{\psi}_{Lm_i}\hat{m}_{n_j}^{m_i}\psi_{R}^{n_j}+\bar{\psi}_{Rm_i}\hat{m}_{n_j}^{m_i}\psi_{L}^{n_j}\\ &\quad-\frac{1}{4\pi\hbar}\{M_{(2)ab}^2W_{\lambda}^{a}W^{b\lambda}+2M_{(1,2)a}^2W_{\lambda}^{a}B^{\lambda}+M_{(1)}^2B_{\lambda}B^{\lambda}\}(1+\varphi)\Bigg]\end{aligned} \tag{3.7}$$

wherein

$$M^2=-2\mu^2\hbar^2 \qquad (\mu^2<0) \tag{3.7a}$$

is the square of the mass of the Higgs field ($\varphi$ field) and

$$\hat{m}_{n_j}^{m_i}=\frac{1}{2}kv(N^{\dagger}\hat{x}_{n_j}^{m_i}+\hat{x}_{n_j}^{\dagger m_i}N) \tag{3.8}$$

is the mass matrix of the fermionic $\psi$ fields, which must be adjusted to the observed mass values of the fermions. The matrices of the mass squares of the gauge fields are defined by

$$M_{(2)ab}^2=4\pi\hbar v^2g_2^2N^{\dagger}\tau_{(a}\tau_{b)}N=M_W^2\delta_{ab} \tag{3.9a}$$

$$M_{(1,2)a}^2=4\pi\hbar v^2g_1g_2\frac{1}{2}N^{\dagger}\tau_aN=-M_W^2\frac{g_1}{g_2}\delta_a^3 \tag{3.9b}$$

$$M_{(1)}^2=\pi\hbar v^2g_1^2=M_W^2\left(\frac{g_1}{g_2}\right)^2 \tag{3.9c}$$

where $N=\binom{0}{1}$ is chosen and

$$M_W=(\pi\hbar)^{1/2}vg_2 \tag{3.10}$$

Diagonalization of (3.9a)-(3.9c) yields the four eigenvalues

$$M_W^2; \quad M_W^2; \quad M_Z^2 = \pi\hbar v^2(g_1^2 + g_2^2); \quad 0 \tag{3.11}$$

with the corresponding eigenvectors

$$W_\lambda^1; \quad W_\lambda^2; \quad Z_\lambda = c_W W_\lambda^3 - s_W B_\lambda; \quad A_\lambda = s_W W_\lambda^3 + c_W B_\lambda \tag{3.11a}$$

wherein $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$ ($\theta_W$ is the Weinberg angle). The field strengths belonging to (3.11a) are given by

$$\begin{aligned} F^{\mu\lambda}_{(W^1)} &= F^{1\mu\lambda}_{(2)}; \qquad F^{\mu\lambda}_{(W^2)} = F^{2\mu\lambda}_{(2)} \\ F^{\mu\lambda}_{(Z)} &= c_W F^{3\mu\lambda}_{(2)} - s_W F^{\mu\lambda}_{(1)} \\ F^{\mu\lambda}_{(A)} &= s_W F^{3\mu\lambda}_{(2)} + c_W F^{\mu\lambda}_{(1)} \end{aligned} \tag{3.12}$$

Herewith we obtain from (3.6a) and (3.6b), in view of (3.9a)-(3.11), the gauge-field equations:[3]

$$D_\mu F^{\mu\lambda}_{(W^{1,2})} + (1+\varphi)^2 \left(\frac{M_W}{\hbar}\right)^2 W^{1,2\lambda} = 4\pi j^{1,2\lambda}_{(2)}(\psi) \tag{3.13a}$$

$$D_\mu F^{\mu\lambda}_{(Z)} + (1+\varphi)^2 \left(\frac{M_Z}{\hbar}\right)^2 Z^\lambda = 4\pi j^\lambda_{(Z)}(\psi) \tag{3.13b}$$

$$D_\mu F^{\mu\lambda}_{(A)} = 4\pi j^\lambda_{(A)}(\psi) \tag{3.13c}$$

with the matter current densities corresponding to (3.12):

$$j^\lambda_{(Z)}(\psi) = c_W j^{3\lambda}_{(2)}(\psi) - s_W j^\lambda_{(1)}(\psi) \tag{3.14a}$$

$$j^\lambda_{(A)}(\psi) = s_W j^{3\lambda}_{(2)}(\psi) + c_W j^\lambda_{(1)}(\psi) \tag{3.14b}$$

In the same way we find from (3.7) for the Higgs field $\varphi$

$$\begin{aligned} &\partial_\mu \partial^\mu \varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2 + \varphi^3) \\ &= -\frac{1}{v^2}\Bigg[\bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R + \bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L \\ &\quad - \frac{1}{4\pi\hbar}\{M_W^2(W^1_\lambda W^{1\lambda} + W^2_\lambda W^{2\lambda}) + M_Z^2 Z_\lambda Z^\lambda\}(1+\varphi)\Bigg] \end{aligned} \tag{3.15}$$

Obviously, in the field equations (3.5a), (3.5b), (3.13a)-(3.13c), and (3.15) the Higgs field $\varphi$ plays the role of an (attractive) scalar gravitational potential between the *massive* particles: According to equation (3.15), the source of $\varphi$ is the mass of the fermions and of the gauge bosons $W^{1,2}$ and $Z$,[4] whereby this equation linearized with respect to $\varphi$ is a potential equation

[3]The covariant derivative in (3.13a)-(3.13c) is defined by the covariant derivative of the right-hand side of (3.12) according to (2.9a).

[4]The second term on the right-hand side of equation (3.15) is positive with respect to the signature of the metric.

of Yukawa type. Accordingly, the potential $\varphi$ has a finite range

$$l = \hbar / M \tag{3.16}$$

given by the mass of the Higgs particle, and $v^{-2}$ has the meaning of the gravitational constant, so that

$$v^{-2} = 4\pi G\gamma \tag{3.17}$$

is valid, where $G$ is the Newtonian gravitational constant and $\gamma$ a dimensionless factor, which compares the strength of the Newtonian gravity with that of the Higgs field and which can be determined only experimentally; see Section 5. On the other hand, the gravitational potential $\varphi$ acts back on the mass of the fermions and of the gauge bosons according to the field equations (3.5a), (3.5b), and (3.13a)-(3.13c). Simultaneously, the equivalence between inertial and passive as well as active gravitational mass is guaranteed. This feature results from the fact that by the symmetry breaking only *one* type of mass is introduced. Evidently, the neutrinos $\nu_L^i$ and the photon $A$ do not participate in this gravitational interaction.

## 4. GRAVITATIONAL FORCE AND POTENTIAL EQUATION

First we consider the potential equation from a more classical standpoint. With respect to the fact of a *scalar* gravitational interaction, we rewrite equation (3.15) with the help of the trace of the energy-momentum tensor, because this should be the only source of a scalar gravitational potential within a Lorentz-covariant theory. From (2.12) and (2.12a)-(2.12c) one finds after symmetry breaking

$$T^\mu_\lambda = T^\mu_\lambda(\psi) + T^\mu_\lambda(W, Z, A) + T^\mu_\lambda(\varphi) \tag{4.1}$$

with $T^\mu_\lambda(\psi)$ given by (2.12a) and

$$\begin{aligned} T^\mu_\lambda(W, Z, A) = T^\mu_\lambda(F) + \frac{1}{4\pi\hbar}\Bigg[ & M_W^2\Big\{(W^1_\lambda W^{1\mu} + W^2_\lambda W^{2\mu}) \\ & -\frac{1}{2}\delta^\mu_\lambda (W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha})\Big\} \\ & + M_Z^2\Big\{Z_\lambda Z^\mu - \frac{1}{2}\delta^\mu_\lambda Z_\alpha Z^\alpha\Big\}\Bigg] \end{aligned} \tag{4.1a}$$

[$T^\mu_\lambda(F)$ according to (2.12b)] as well as

$$T^\mu_\lambda(\varphi) = v^2\left[\partial_\lambda\varphi\,\partial^\mu\varphi - \frac{1}{2}\delta^\mu_\lambda\left\{\partial_\alpha\varphi\,\partial^\alpha\varphi + \frac{M^2}{4\hbar^2}(1+\varphi)^2(1-2\varphi-\varphi^2)\right\}\right] \tag{4.1b}$$

From this it follows immediately, using the field equations (3.5a) and (3.5b) that

$$T = T^{\lambda}_{\lambda} = T(\psi) + T(W, Z, A) + T(\varphi) \tag{4.2}$$

with

$$T(\psi) = [\bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R + \bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L](1+\varphi) \tag{4.2a}$$

$$\begin{aligned} T(W, Z, A) &= T(W, Z) \\ &= -\frac{1}{4\pi\hbar}[M^2_W(W^1_\lambda W^{1\lambda} + W^2_\lambda W^{2\lambda}) + M^2_Z Z_\lambda Z^\lambda](1+\varphi)^2 \end{aligned} \tag{4.2b}$$

and

$$T(\varphi) = v^2\left[\frac{M^2}{2\hbar^2}(\varphi^4 + 4\varphi^3 + 4\varphi^2 - 1) - \partial_\lambda \varphi\, \partial^\lambda \varphi\right] \tag{4.2c}$$

In the appendix it is shown that $T(\psi)$ separates, in total analogy to $T(W, Z)$, into the masses of the single fermions:

$$T(\psi) = \sum_i (m_{e^i} \bar{e}_i e^i + m_{u^i} \bar{u}_i u^i + m_{d^i} \bar{d}_i d^i)(1+\varphi) \tag{4.2a$'$}$$

Comparing (4.2a) and (4.2b) with the right-hand side of the Higgs-field equation (3.15), one finds that the source of the potential $\varphi$ is given by the first two terms of the trace (4.2). In this way we find, using (3.17),

$$\begin{aligned} \partial_\mu \partial^\mu \varphi + \frac{M^2}{\hbar^2}\varphi + \frac{1}{2}\frac{M^2}{\hbar^2}(3\varphi^2 + \varphi^3) \\ = -4\pi G\gamma(1+\varphi)^{-1}[T(\psi) + T(W, Z)] \end{aligned} \tag{4.3}$$

In the linearized version (with respect to $\varphi$), equation (4.3) represents a potential equation for $\varphi$ of Yukawa type with the trace of the energy-momentum tensor of the massive fermions and the massive gauge bosons $W^{1,2}$ and $Z$ as source.

Finally, we investigate the gravitational force caused by the Higgs field in more detail. Insertion of the symmetry breaking according to (3.1)–(3.4a) into the first integral of the right-hand side of (2.15) yields

$$\begin{aligned} K_\lambda &= k[(D_\lambda \phi)^\dagger \bar{\psi}_{Rm_i} \hat{x}^{m_i}_{n_j} \psi^{n_j}_L + \bar{\psi}_{Lm_i} \hat{x}^{\dagger m_i}_{n_j} \psi^{n_j}_R D_\lambda \phi] \\ &= (\bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L + \bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R)\partial_\lambda \varphi \\ &\quad + v(1+\varphi)[(D_\lambda N)^\dagger k\bar{\psi}_{Rm_i} \hat{x}^{m_i}_{n_j} \psi^{n_j}_L + k\bar{\psi}_{Lm_i} \hat{x}^{\dagger m_i}_{n_j} \psi^{n_j}_R D_\lambda N] \end{aligned} \tag{4.4}$$

Substitution of the conglomerate $k\bar{\psi}_{Rm_i} \hat{x}^{m_i}_{n_j} \psi^{n_j}_L$ by the left-hand side of the field equation (2.11) results, with the use of (3.3a) and (3.4a), in

$$\begin{aligned}
K_\lambda = \Bigg\{ & \bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R + \bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L \\
& - \frac{1}{4\pi\hbar} [M_W^2 (W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha}) + M_Z^2 Z_\alpha Z^\alpha](1+\varphi) \Bigg\} \partial_\lambda \varphi \\
& - \frac{1}{4\pi\hbar} \partial_\mu \Bigg[ (1+\varphi)^2 \Bigg\{ M_W^2 \Bigg[ W^1_\lambda W^{1\mu} + W^2_\lambda W^{2\mu} \\
& - \frac{1}{2} \delta^\mu_\lambda (W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha}) \Bigg] + M_Z^2 \left( Z_\lambda Z^\mu - \frac{1}{2} \delta^\mu_\lambda Z_\alpha Z^\alpha \right) \Bigg\} \Bigg] \\
& + i \frac{v^2}{2} (1+\varphi)^2 \{ g_2 F^a_{(2)\mu\lambda} [N^\dagger \tau_a D^\mu N - (D^\mu N)^\dagger \tau_a N] \\
& + \frac{1}{2} g_1 F_{(1)\mu\lambda} [N^\dagger D^\mu N - (D^\mu N)^\dagger N] \}
\end{aligned} \tag{4.5}$$

By insertion of (4.5) into the right-hand side of (2.15), the last brackets of (4.5) and (2.15) cancel out, whereas the second bracket of (4.5) can be combined with $\partial_\mu T^\mu_\lambda(F)$ to $\partial_\mu T^\mu_\lambda(W, Z, A)$ according to (4.2b). In this way we obtain, neglecting surface integrals, at the spacelike infinity

$$\begin{aligned}
& \frac{\partial}{\partial t} \int [T^0_\lambda(\psi) + T^0_\lambda(W, Z, A)]\, d^3x \\
& = \int \Bigg[ \bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R + \bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L \\
& \quad - \frac{1}{4\pi\hbar} \{ M_W^2 (W^1_\alpha W^{1\alpha} + W^2_\alpha W^{2\alpha}) + M_Z^2 Z_\alpha Z^\alpha \} (1+\varphi) \Bigg] \partial_\lambda \varphi \, d^3x
\end{aligned} \tag{4.6}$$

In total analogy to the procedure yielding the potential equation (4.3), we substitute the bracket of the 4-force in (4.6) by the traces $T(\psi)$ and $T(W, Z)$ given by (4.2a) and (4.2b), respectively; so we find

$$\begin{aligned}
& \frac{\partial}{\partial t} \int [T^0_\lambda(\psi) + T^0_\lambda(W, Z, A)]\, d^3x \\
& = \int (1+\varphi)^{-1} [T(\psi) + T(W, Z)] \partial_\lambda \varphi \, d^3x
\end{aligned} \tag{4.7}$$

Considering the transition from equation (2.15) to (2.17), we can express the time derivative of the 4-momentum of the gauge fields by a 4-force acting on the fermionic matter currents. Restricting this procedure to the *massless* gauge field $A^\lambda$ (photon), we get from (4.7)

$$\frac{\partial}{\partial t}\int [T^0_\lambda(\psi)+T^0_\lambda(W,Z)]\,d^3x$$

$$=\int \hbar F_{(A)\lambda\mu}j^\mu_{(A)}(\psi)\,d^3x+\int (1+\varphi)^{-1}[T(\psi)+T(W,Z)]\partial_\lambda\varphi\,d^3x \quad (4.8)$$

Herein the first term of the right-hand side describes the 4-force of the massless gauge–boson acting on the matter fields, i.e., the electromagnetic Lorentz-force coupled by the electric charge [see (3.14b)]

$$e=s_W g_2=c_W g_1 \quad (4.8a)$$

The second term [identical with the right-hand side of (4.7)] is the attractive gravitational force on the masses of the fermions and of the gauge bosons $W^{1,2}$ and $Z$, which are simultaneously the source of the Higgs potential $\varphi$ according to (4.3). This behavior is exactly that of classical gravity, coupling to the mass (≡energy) only and not to any charge. However, the qualitative difference with respect to the Newtonian gravity consists, besides the non-linear terms in (4.3), in the finite range of $\varphi$ caused by the Yukawa term.

## 5. FINAL REMARKS

In the end we point out some interesting features of our result. First we note that, in view of the right-hand side of (4.7), it is appropriate to define

$$\ln(1+\varphi)=\chi \quad (5.1)$$

as the new gravitational potential, so that the momentum law reads

$$\frac{\partial}{\partial t}\int [T^0_\lambda(\psi)+T^0_\lambda(W,Z,A)]\,d^3x=\int [T(\psi)+T(W,Z)]\partial_\lambda\chi\,d^3x \quad (5.2)$$

Then the nonlinear terms concerning $\varphi$ in (4.3) can be expressed by $T(\varphi)\equiv T(\chi)$ according to (4.2c). In this way the field equation for the potential $\chi$ (excited Higgs field) takes the very impressive form

$$\partial_\mu\partial^\mu e^{2\chi}+\frac{M^2}{\hbar^2}e^{2\chi}=-8\pi G\gamma[T(\psi)+T(W,Z)+T(\chi)] \quad (5.3)$$

Equations (5.2) and (5.3) are indeed those of scalar gravity with self-interaction in a natural manner. For the understanding of the Higgs field it may be of interest that the structure of equation (5.3) exists already before the symmetry breaking. Considering the trace $T$ of the energy-momentum tensor (2.12), one finds, with the use of the field equations (2.8a), (2.8b), and (2.11),

$$\partial_\mu\partial^\mu(\phi^\dagger\phi)+\left(\frac{M}{\hbar}\right)^2(\phi^\dagger\phi)=-2T \quad (5.4)$$

with $M^2 = -2\mu^2\hbar^2$. Accordingly, the Yukawa-like self-interacting scalar gravity of the Higgs field is present within the theory from the very beginning. Equation (5.4) possesses an interesting behavior with respect to the symmetry breaking. From the second term on the left-hand side there results in view of (3.1) in the first step a cosmological constant $M^2v^2/\hbar^2$; but this is compensated exactly by the trace of the energy-momentum tensor of the ground state. In our opinion this is the property of the cosmological constant at all, also in general relativity.

Furthermore, we emphasize that the gravitational action of the Higgs field is not restricted to the Glashow–Salam–Weinberg model, but it is valid in all cases of mass production by symmetry breaking via the Higgs mechanism (Dehnen *et al.*, 1990a), e.g., also in the GUT model. However, because in (3.16) the mass $M$ is that of the Higgs particle, the range $l$ of the potential $\varphi$ should be very short, so that until now no experimental evidence for the Higgs gravity exists, at least in the macroscopic limit. For this reason it also appears improbable that it has something to do with the non-Newtonian gravity currently discussed as the so-called fifth force (Eckhardt *et al.*, 1988).

Finally, the factor $\gamma$ in (3.17) can be calculated from (3.10) by the use of the mass of the $W$ bosons and the value of the gauge coupling constant $g_2$; one finds

$$\gamma = \frac{\hbar g_2^2}{4GM_W^2} = \frac{1}{2} g_2^2 \left(\frac{M_P}{M_W}\right)^2 = 2\times 10^{32} \tag{5.5}$$

($M_P$ is the Planck mass). Consequently, the Higgs gravity represents a relatively strong scalar gravitational interaction between the massive elementary particles, with, however, extremely short range and with the essential property of quantizability. If any Higgs field exists in nature, this type of gravity is present.

The expression (5.5) shows that in the case of a symmetry breaking where the bosonic mass is of the order of the Planck mass, the Higgs gravity approaches the Newtonian gravity if the mass of the Higgs particle is sufficiently small. In this connection the question arises, following Einstein's idea of relativity of inertia, if it is possible to construct a tensorial quantum theory of gravity with the use of the Higgs mechanism, leading at last to Einstein's gravitational theory in the classical macroscopic limit.

## APPENDIX

In order to show the separation of $T(\psi)$ into the single fermionic masses it is necessary to specify the fermionic mass matrix as follows [without suppression of the $SU(2)$ indices $I, J, \ldots$; note that in equation (A1) the sum convention does not hold]:

$$\hat{m}^{m_i}_{n_j} = \hat{m}^{Im_i}_{Jn_j} = \sum_k m_{Im_k} \delta^I_J \delta^m_n (U^{I_m}_{(c)})^i_k (U^{I_m}_{(c)})^{-1k}_j \tag{A1}$$

where

$$(U^{I_m}_{(c)})^i{}_k = \begin{cases} U^i_{(c)k} & \text{if} \quad (I, m) = (d', q) \\ \delta^i_k & \text{otherwise} \end{cases} \tag{A2}$$

with the Cabibbo matrix $U^i_{(c)k}$ according to (2.3b). Insertion of (A1) into the right-hand side of (4.2a) yields

$$\begin{aligned} &\bar{\psi}_{Lm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_R + \bar{\psi}_{Rm_i} \hat{m}^{m_i}_{n_j} \psi^{n_j}_L \\ &\quad = \sum_i [m_{e^i}(\bar{e}_{R_i} e^i_L + \bar{e}_{L_i} e^i_R) + m_{u^i}(\bar{u}_{R_i} u^i_L + \bar{u}_{L_i} u^i_R) \\ &\qquad + m_{d^i}(\bar{d}_{R_i} d^i_L + \bar{d}_{L_i} d^i_R)] \end{aligned} \tag{A3}$$

which immediately goes over into the expression (4.2a′).

The mass-matrix (A1) follows from (3.8) after insertion of the Yukawa coupling matrix

$$\hat{x}^{m_i}_{n_j} = \hat{x}^{RIm_i}_{J_{n_j}} = C^{\dagger R}_{ImS} N^S \delta^I_J \delta^m_n \sum_k x_{I_{m_k}} (U^{I_m}_{(c)})^i{}_k (U^{I_m}_{(c)})^{-1k}_j \tag{A4}$$

where the real quantities $x_{I_{m_k}}$ contain the masses

$$m_{Im_k} = \frac{kv}{2} x_{I_{m_k}} \{N^\dagger (C^\dagger_{Im} + C_{Im}) N\} \tag{A5}$$

and $C_{Im}$ is given by

$$C^R_{ImS} = \begin{cases} C\delta^R_S \text{ for } (I, m) = (u, q), \\ \delta^R_S \qquad\qquad \text{otherwise} \end{cases} \tag{A6}$$

with

$$C' = e^{i\lambda} C \tag{A7}$$

under hypercharge-transformation $e^{i\lambda Y}$ (hypercharge-conjugation-operator).

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